Lectures on Electromagnetic theory I

PH 2151

Lecture 5
(The vector operator ∇ or del operator)

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The del operator ∇ and ∇.D in cartesian coordinate system

$$\boldsymbol{\nabla} = \frac{\partial}{\partial x}\boldsymbol{a_x} + \frac{\partial}{\partial y}\boldsymbol{a_y} + \frac{\partial}{\partial z}\boldsymbol{a_z}$$

D $= D_x\boldsymbol{a_x} + D_y\boldsymbol{a_y} + D_z\boldsymbol{a_z}$

$$(\boldsymbol{div\ D}) \quad \boldsymbol{\nabla}.\boldsymbol{D} = \frac{\boldsymbol{\partial D}}{\boldsymbol{\partial x}} + \frac{\boldsymbol{\partial D}}{\boldsymbol{\partial x}} + \frac{\boldsymbol{\partial D}}{\boldsymbol{\partial x}}$$

Ex: If D = 4xy$\boldsymbol{a_x}$ – $xy^2\ \boldsymbol{a_y}$ + 5 sin z $\boldsymbol{a_z}$

Find $\boldsymbol{\nabla}.\boldsymbol{D}$ at (2,2,0)

The ∇.D in cylindrical coordinates

If $h_\rho = 1$, $h_\emptyset = \rho$, h_z =1

Prove that: ∇.D = $\frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho D_\rho) + \frac{1}{\rho}\frac{\partial D_\emptyset}{\partial\emptyset} + \frac{\partial D_z}{\partial z}$

Ex :

If D = $\rho\, sin\emptyset\, \boldsymbol{a}_\rho + 2\rho\, cos\emptyset\, \boldsymbol{a}_\emptyset + 2z^2 a_z$

Find $\nabla.D$

The ∇.D in spherical Coordinates

- If $h_r = 1$, $h_\theta = r$, $h_\emptyset$=r sinθ , Prove that :

- $\boldsymbol{\nabla} . \boldsymbol{D} = \frac{\mathbf{1}}{\boldsymbol{r^2}} \frac{\boldsymbol{\partial}}{\boldsymbol{\partial r}} (\boldsymbol{r^2 D_r}) + \frac{\mathbf{1}}{\boldsymbol{r \, sin\theta}} \frac{\boldsymbol{\partial}}{\boldsymbol{\partial \theta}} (\mathbf{sin}\boldsymbol{\theta D_\theta}) + \frac{\mathbf{1}}{\boldsymbol{r \, sin\theta}} \frac{\boldsymbol{\partial D_\emptyset}}{\boldsymbol{\partial \emptyset}}$

- Ex: If D = r$\boldsymbol{a_r}$ - $r^2 \, cot\theta \boldsymbol{a_\theta}$,find $\nabla . D$

Problems

1) Prove that divE for a line charge equals zero except at $\rho = 0$.,where E electric field intensity.
2) If A=$5x^2\ (sin\frac{\pi x}{2})a_x$, $find\ \nabla . A$ at x=1
3) Prove that divD for a point charge equals zero , where D is the electric flux density, except at r=0
4) If A=$\rho\ sin\emptyset\ \boldsymbol{a_\rho} + \rho^2 cos\emptyset\ \boldsymbol{a_\emptyset} + 2\rho e^{-5z}\boldsymbol{a_z}$,find divA at $(\frac{1}{2}, \pi/2, 0)$
5) If A=$(\frac{5}{r^2})\ sin\theta\boldsymbol{a_r} + r\ cot\theta\boldsymbol{a_\theta}$ + r sin$\theta\ cos\emptyset\boldsymbol{a_\emptyset}$,find divA.